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4D-Space-Time Geometry & Cosmological Constant

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ABSTRACT

In this paper, 4-dimensional space-time geometry has been discussed. The smallness of the effective cosmological constant constitutes the most difficult problems involving cosmology. Recent observations of Type Ia supernovae and measurements of the cosmic microwave background suggest that the universe is in an accelerating expansion phase.

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1-INTRODUCTION:

In 1880 ***Charles Howard Hinton's*** has written his famous article *what is the fourth dimension?* [1] in reply to it, in the journal ***nature*** in the year 1885, an anonymous letter writer signing **"s",** *introduced time as the fourth dimension* and dealt with a 4-dimensional ***"time-space".*** s. mastered what we now call the *space-time* picture, and even managed to correctly describe the hypercube by looking at the motion of a cube in time-space[2]. It was made clear that time is not considered as a ***fourth space like dimension*** as the German translation of wells book came out in 1904 [3], but a philosopher ***Menyh´ert (melchior)*** concluded this debate in his published work ***"new theory of space and time"*** in 1901 [1], he has given the name to a 4-dimensional entity as **"flowing space",** *according to him* "the coordinates of a point in flowing space could be represented by $x + it, y + it, z + it$ [3].

In sharp contrast, around 1905, and before Minkowski, Poincar´e also had a 4-dimensional (space-time) formalism for the wave equation and electrodynamics [4]. Johannes kepler who had related physical bodies, the planets, to geometric objects, i.e., to the five regular polyhedral, certainly was far from what we now understand by geometrization of physics, i. e. the embedding of physical objects (matter, fields) into a geometrical framework. A weakening of the rigid understanding of space seems to have occurred when the notion of non-Euclidean geometry came up, in the 19th century. The suitable answer to the question of what kind of geometry, the space we live is under investigation from long time.

2-SCHWARZSCHILD INVESTIGATION

Schwarzschild investigated the question scientifically with bodies far away in the heavens (1900) [5]. Also in the 19th century, the mechanics of rigid bodies reformulated within non-Euclidean geometry, with the exception of clifford, Latter on scientific community said it as ***space-time*** by ***Hermann minkowski*** after famous speech about the ***"union of space and time***" [6]. A few mathematicians, fiction writers, and philosophers presented it quite clearly before Minkowski, but not as a mathematical theory.

The mathematical formulation of the spatial homogeneity and isotropy of the universe gives the following results:

(i). The hyper surfaces with constant cosmic time are maximally symmetric subspaces of the whole space time.

(ii).Not only the metric g_{ij} but all cosmic tensors such as the energy momentum tensor T_{ij} , are invariant w. r. t. isometery of subspaces.

iii) The metric of space with homogeneous & isotropic sections which being maximally symmetric, is the Robertson Walker Metric (R.W). This may be explained as:

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \quad (1)$$

where $R(t)$ is an unknown function of time. This sets the scale of the geometry of the space, therefore is called the ***scale factor***. k is a constant and may be chosen to be ***-1, 0, 1***. The coordinates (r,θ,ϕ,t) form a commoving coordinate system in the sense that the fundamental particles are at rest w. r. t. (r,θ,ϕ) . Now we wish to discuss the ***EFE*** (Einstein Field Equations)

3- EINSTEIN FIELD EQUATIONS

The Einstein field equations (**EFE**) or Einstein's equations are a set of 10 equations in Albert Einstein's General theory of Relativity which describe the fundamental interaction of gravitation as a result of space-time being curved by matter and energy. First time the result was published by Einstein himself in 1915 as a tensor equation, the EFE equate local space-time curvature with the local energy and momentum within that space-time as discussed in detail below:

Einstein Field Equations describes the gravitational field resulting from the distribution of matter in the universe. It is expressed as:

$$R_{ij} - \frac{1}{2} R g_{ij} = -\frac{8\pi G}{c^4} T_{ij} \quad (2)$$

where R_{ij} is the Ricci tensor, R is the curvature scalar, T_{ij} is the energy momentum tensor of the source producing the gravitational field and G is the Newton's Gravitational constant.

The Einstein field equations are used to determine the curvature of the space time resulting from the presence of mass & energy. Because of that they determine the metric tensor of the space time for a given arrangement of stress energy in the space time. The energy momentum tensor satisfies the following conditions:

i) T_{ij} is symmetric w. r. t. interchange of i & j.

ii) T_{ij} is divergence-less for energy and momentum to be conserved.

i.e. $$T^{ij}_{;\ i} = 0 \quad , \qquad (i,\ j = 1,2,3,4) \tag{3}$$

Here, $$T^{ij} = \rho u^i u^j \tag{4}$$

The symmetric tensor T^{ij} is the material energy momentum tensor.

we can define

$$\begin{aligned} T^{ij}{}_{;i} &= (\rho u^i u^j)_{;j} \\ &= (\rho u^j)_{;j} u^i + \rho u^j (u^i{}_{;j}) \\ &= \rho u^j u^i{}_{;j} \end{aligned} \tag{5}$$

If, u^j is regarded as a field function (i.e. meaningful not just on one world line but a whole set of worldliness filling up all space time or a region thereof), we obtain

$$\frac{du^j}{ds} = \left(\frac{\partial u^j}{\partial x^j}\right)\left(\frac{dx^j}{ds}\right) = u^i{}_{;j} u^j \tag{6}$$

we get by using other relations

$$\begin{aligned} u^i{}_{;j} u^j + \Gamma^i_{jk} u^j u^k &= \left(u^i{}_{;j} + \Gamma^i_{jk} u^k\right) u^j \\ &= u^i{}_{;j} u^j = 0 \end{aligned} \tag{7}$$

with the use of (1.5) and (1.7), we can get

$$T^{ij}_{;\ i} = 0$$

So that the tensor T^{ij} can be used in the right hand side of Einstein equations. However the tensor given by $T^{ij} = \rho u^i u^j$ is a special case, which occurs in

$$T^{ij} = (\rho + p) u^i u^j - p g^{ij} \tag{8}$$

here ρ is the mass energy density being obtained from the later by setting $p = 0$. This zero-pressure case obtains when there is no random motion of the material particles that is associated with pressure, so that the particles move solely under the influence of gravitation and so move along geodesics

$$\frac{du^k}{ds} + \Gamma^k_{ij} u^i u^j = 0 \tag{9}$$

The Ricci tensor is defined by:

$$R_{ij} = \frac{\partial^2 \log\sqrt{-g}}{\partial x^i \partial x^j} - \frac{\partial \Gamma^k_{ij}}{\partial x^k} + \Gamma^l_{im}\Gamma^m_{jl} - \Gamma^k_{ij}\frac{\partial \log\sqrt{-g}}{\partial x^k} \tag{10}$$

and $$R = g^{ij} R_{ij} \tag{11}$$

where g_{ij} is the metric tensor & g is its determinant. Γ^i_{jk} are Cristoffel symbols related to g_{ij}. The constant $\frac{8\pi G}{c^4}$ on the right hand side of the equation is obtained by the weak & static field approximation of the equation (1.2) and then comparing it with Poisson's equation:

$$\nabla^2 \phi = 4G\pi\rho \quad (12)$$

governing the gravitational field in Newtonian mechanics. ϕ is the gravitational potential & ρ is the density of the matter.

4- DIRAC APPROACH FOR VARIABLE CONSTANT OF GRAVITATION '*G*'

We wish to bring the attention of the reader regarding Dirac approaches leading towards a variable constant of Gravitation. This approach is known as the Dirac large number hypothesis [7]. The ratio of the gravitational to the electrostatic force is of the order 10^{-40}. There is no convincing explanation of why such a small dimensionless number appears in the fundamental laws of physics. Dirac pointed out that a dimensionless no. of the order of 10^{-40} may be constructed with *G*, *h*, *c* and the Hubble's constant H_0. It means if H is not a constant due to the expansion of the universe then the constant G may also vary with time.

5- REGARDING FUNDAMENTAL "CONSTANTS"

The discrepancy of fundamental "constants" is one of the most spectacular and unsettled problems in cosmology. The Einstein's field equations (with $c = 1$) necessitate two of such constants viz., the gravitational constant (G) and the cosmological constant (Λ), where G plays the role of coupling constant between geometry and matter while, Λ was introduced by Einstein in 1917 as the universal repulsion to make the universe static in accordance with generally accepted picture of that time. Subsequently, a general expansion of the universe was observed by Hubble in 1927. Recent observations also incertitude the stability of fundamental constants and "Equivalence Principle of General Relativity.

As stated in the above section 4, Dirac [8-9] was first to introduce the time variation of the gravitational constant G in his large number hypothesis and since then it has been used frequently in numerous modifications of General theory of Relativity. G has many interesting consequences in astrophysics. It is shown that G-varying cosmology is consistent with whatsoever cosmological observations available at present [10].

6-CONCLUDING REMARKS:

The problem of the cosmological constant is one of the most salient and unsettled problems in cosmology so one of the basic purpose of this paper is to draw the attention of young researchers on this topic where a lot of unaddressed problem is waiting for the suitable reply in the world of work. The smallness of the effective cosmological constant recently observed $(\Lambda_0 \leq 10^{-56}\text{cm}{-2})$ constitutes the most difficult problems involving cosmology and elementary particle physics theory. To discuss the striking cancellation between the "bare" cosmological constant Λ & ordinary vacuum energy contributions of the quantum fields, many suitable ways have been proposed during last few years [11]. This problem may be expressed as the discrepancy between the minimum value of Λ for the present universe. The values 10^{50} larger expected by the Glashow-Salam-Weinberg model [12] or by grand unified theory (GUT) where it should be 10^{107} larger [13]. The cosmological constant Λ is then smaller value at the present epoch. Many cosmological models with variable G and variable cosmological constant have been constructed and published by several authors[14-18] in search of exact mystery regarding expansion of the universe. This is one of the important areas of the research now in these days. Recent observations of Type Ia supernovae [19−22] and measurements of the cosmic microwave background [23] suggest that the universe is in an accelerating expansion phase.

REFERENCES

[1]. Charles Howard Hinton. "What is the fourth dimension?" Scientific Romances No. 1. London: Swan Sonnenschein & Co. (1884).

[2]. S. "Four-Dimensional Space." Nature 31, No. 804, March 26, 481. (1885).

[3]. H. G. Wells. "Die Zeitmaschine." Deutsch v. Felix P. Greve (Frederic Ph. Grove). Minden: Brund (1904).

[4]. Henri Poincar´e."Sur la dynamique de l'´electron."Comptes rendus de l' Academie des Sciences, Paris 140, 1504-1508 (1905).

[5]. Karl Schwarzschild. "Uber das zul¨assige Kr¨ummungsmaß des Raumes." Vierteljahresschrift der astronomischen Gesellschaft (Leipzig) 35, 337- 347 (1900).

[6]. Hermann Minkowski. "Raum und Zeit." Vortrag gehalten auf der 80. Naturforscher-Versammlung zu K¨oln am 21. September 1908. Leipzig und Berlin: B. G. Teubner (1909).

[7]. Weinberg, S. (1972), 'Gravitation & Cosmology' *John Willy & Sons*, pp. 409.

[8]. Ya.B. Zeldovich, “The equation of state at ultrahigh densities and its relativistic limitations”, Soviet Physics-JETP, vol. 14, no. 5, pp. 1143, 1962.

[9]. J.D. Barrow, “Quiescent cosmology”, Nature, volume 272, no. 5650, pp. 211-215, 1978.

[10]. M.A.H. MacCallum, “A class of homogeneous cosmological models III: asymptotic behaviour”, Communication of Mathematical Physics, vol. 20, no. 1, pp. 57-84, 1971.

[11]. C.B. Collins, “Global structure of the Kantowski Sachs cosmological models”, Journal of Mathematical Physics, vol. 18, no. 11, pp. 2116-2124, 1977.

[12]. V. Sahni, T.D. Saini, A.A. Starobinsky and U. Alam, “Statefinder-A new geometrical diagnostic of dark energy”, Journal of Experimental and Theoretical Physics (JETP) Letters, vol. 77, no. 5, pp. 201-206, 2003.

[13]. U. Alam, V. Sahni, T.D. Saini and A.A. Starobinsky, “Exploring the expanding Universe and dark energy using the statefinder diagnostic”, Monthly Notices of the Royal Astronomical Society, vol. 344, issue 4, pp. 1057-1074, 2003.

[14]. Pande, H. D., Chandra, R. and Mishra, Ravi Kant, (1997), ‘Variation of G and Λ in Homogeneous and Isotropic Cosmological Models’, *J. Nat. Acad. Math.*, Vol. 11, pp. 118-12.

[15]. Pande, H. D., Chandra, R. and Mishra, Ravi Kant, (2000), ‘Cosmological Models with variable Cosmological and Gravitational constant’, *Indian J. pure appl. Math.*, 31(2), pp. 161.

[16]. Chawla, Chanchal, Mishra R.K**,** 2012, “*Bianchi Type-I Viscous fluid cosmological models with variable deceleration parameter”* Romanian Journal of Physics (RJP).

[17]. Mishra R.K, Chawla, Chanchal, 52, 2013, **ISSN 00207748,** Anisotropic Viscous Fluid Cosmological Models from Deceleration to Acceleration in String Cosmology ***International Journal of Theoretical physics*** (IJTP).

[18]. Chawla, Chanchal, Mishra R.K, Pradhan A. 2012,127:137,String cosmological models from early deceleration to current acceleration phase with varying G and Λ, ***Eur. Phys. J. Plus.***

[19]. J. Maga~na, T. Matos, V.H. Robles and A. Su_arez, “A brief review of the scalar field dark matter model” arXiv:1201.6107[astro-ph.CO], 2012. 914

[20]. P A M Dirac *Nature* 139 323 (1937).

[21]. P A M Dirac *Nature* 139 1001 (1937).

[22]. V M Canuto and J V Narlikar *Astrophys. J.* 236 6 (1980).

[23]. S Weinberg *Rev. Mod. Phys.* 61 1 (1989).